NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

MATHEMATICS with CALCULUS

Level 3

90636 Integrate functions and solve problems by integration, differential equations or numerical methods

Credits: Six

Answer ALL questions in the spaces provided in this booklet.

Show ALL working for ALL questions.

Check that this booklet has pages 2–9 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only **Achievement Criteria**		
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Integrate functions and solve problems by integration, differential equations or numerical methods. ☐	Find integrals and use integration to solve problems. ☐	Use a variety of integration techniques to solve problem(s). ☐
Overall Level of Performance ☐		

You are advised to spend 50 minutes answering the questions in this booklet.

QUESTION ONE

Find the integrals. You do not need to simplify your answers.

Do not forget the arbitrary constants.

(a) $\int (3\sqrt{x} + \frac{4}{\sqrt{x}})\, dx$

(b) $\int e^{2x+1}\, dx$

(c) $\int (\frac{4x+3}{x})\, dx$

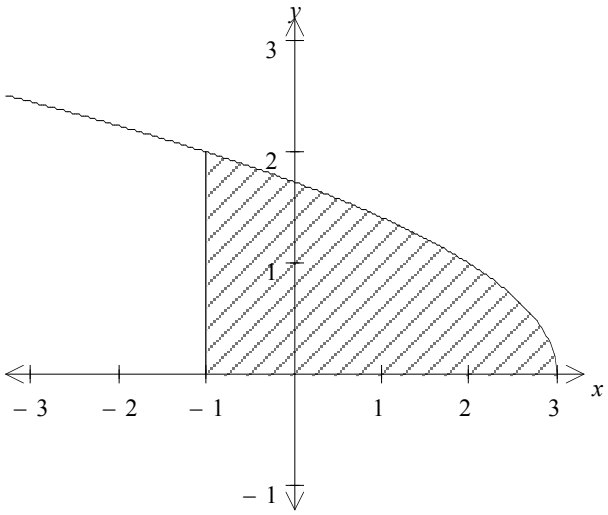
QUESTION TWO

Solve the differential equation $\frac{dy}{dx} = \frac{x}{\cos y}$ evaluating any constants,

given that $y = \frac{\pi}{2}$ when $x = 0$. There is no need to rearrange the solution after integrating.

QUESTION THREE

The graph shown is $y = \sqrt{3 - x}$.



The shaded area is rotated 360°about the x-axis. Find the volume of the solid created.

QUESTION FOUR

Environment Waikato needed to find the volume of water in Rugged River.
The first step to do this involved finding the area of the cross section of the river.
The table shows the depth of the river at 2 metre intervals across the river.

Distance across Rugged River (m)	**0**	**2**	**4**	**6**	**8**	**10**	**12**
Depth of River (m)	**0**	**0.8**	**1.2**	**1.9**	**2.1**	**1.1**	**0**

Use Simpson's Rule to find the estimated area of the cross section of Rugged River.

QUESTION FIVE

Find the integrals.

(a) $\int_0^{\frac{\pi}{4}} (6\sin x \cos x)\ \mathrm{d}x$

(b) $\int \frac{6x}{\sqrt{3x+4}}\ \mathrm{d}x$ The substitution $u = 3x + 4$ may be of help.

QUESTION SIX

A loud hailer component is modelled by rotating the part of the graph $y = \frac{1}{2}e^x$ between $x = \frac{1}{2}$ and $x = 3$ about the x-axis.

Find the volume of this loud hailer component.

QUESTION SEVEN

Find the area enclosed between the curves $y = x^3 + 1$ and $y = x + 1$.

QUESTION EIGHT

The new town reservoir holds 100 million litres of water and supplies the town with 1 million litres of water per day.

The reservoir is partly filled by a spring which provides 0.9 million litres per day, and the rest of the water comes from run-off from the surrounding land.

The spring has no salt but the run-off contains salt with a concentration of 0.00002 kg/L.

Assume that there was no salt in the reservoir originally, that the reservoir is well mixed (ie the water supplied to the town contains the concentration of salt in the reservoir at that instant) and that the reservoir remains full.

(a) Write a differential equation for $\frac{\mathrm{d}Q}{\mathrm{d}t}$ where Q is the quantity of salt, in kg, in the reservoir at any stage and t is the time, in days.

(b) Solve this differential equation to find Q, the quantity of salt in the reservoir, at any time t.